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# *Mathematical Discourse* in Henri Poincaré and Anna Sfard: A Comparison

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**Abstract:** This paper aims to compare the notion of *discours* in Henri Poincaré and that of *discourse* in Anna Sfard's theory of *commognition*, with the aim of clarifying the epistemological differences that distinguish them. In particular, we will investigate how, in Poincaré, *discours* constitutes the condition for the objectivity of science: only what can be transmitted through a shared language can be considered knowledge. *Mathematical discours* is a rigorous tool that transforms facts in the rough into scientific facts. The theory of *commognition* reformulates Poincaré's concept of *discours* in a communicative key, departing from its cognitive rigour.

A synoptic reading of the founding texts of the two authors, separated by a century of history and philosophy of science, allows us to highlight the points of contact and to note the discrepancies.

**Keywords:** Poincaré, Sfard, commognition, mathematical discourse, epistemology, mathematics education.

## § 1. — Introduction.

In this work, we propose a synoptic reading of Poincaré's epistemological writings and the modern theory of *commognition*, mainly due to Anna Sfard.

We consider this comparison important for two reasons: a correct philosophical and epistemological positioning of *commognition*, and a reconstruction of its sources.

The idea, which in the context of 20th-century pedagogy goes back mainly to Vygotsky, that culture is a product of social relations, is translated into mathematics teaching in a learning model that is almost exclusively the result of social relations, between peers or with the teacher, within the classroom. In this context, Sfard's work tends to demonstrate that what the school environment must achieve is the creation of a shared language (called *mathematical discourse*), which is indispensable for the growth of students' mathematical culture. This language can be constructed spontaneously within the classroom, but the role of the teacher is ultimately to lead students towards the official mathematical language.

Let us recall here the essential features of Anna Sfard's theory. *Commognition* is a neologism, the result of the fusion of the terms *communication* and *cognition*. According to this theory, learning cannot be separated from communication, since thought itself is a communicative activity with oneself. Mathematics is a discursive activity and learning it is the process of individualising the *discourse* of the expert; this process is characterised by the use of *specific words*, *visual mediators*, *approved narratives* and *routines*. Mathematics is also a particular discourse that distinguishes it from other discourses in that it is an autopoietic system: unlike other disciplines, in which objects are entities separate from discourse, mathematical objects are discursive constructs, whose creation (*objectification*) occurs by combining three discursive operations: *reification*, *saming* and *encapsulating*.[(1)]

Alongside some undoubtedly very interesting ideas, the position of *commognition* tends to favour teaching practices whose effectiveness deserves to be evaluated in some detail, as will be discussed later.

In particular, our attention turns to the notion of *mathematical discourse*, which is also present in Poincaré's philosophical work.

[(1)] A. Sfard, *Thinking as communicating*, Cambridge University Press, 2008.

It is therefore important to compare the epistemological proposal regarding *mathematical discours* of the great French mathematician with the interpretation proposed in terms of *commognition*. We will try to show that Poincaré's vision, although textually very close to the definition of *mathematical discourse* accepted in the field of *commognition*, is rather disjointed by the latter, reducing its rigour and applicability to the mathematical context.

We will also discuss a reconstruction of the sources of the theory of *commognition*. To our knowledge, although Poincaré is frequently mentioned in the works of Sfard and her admirers, his fundamental text proposing the concept of *mathematical discours*, *La valeur de la science*, is never cited in their works. We believe it is important, in order to reflect correctly on the strengths and weaknesses of the theory of *commognition*, to bear in mind the characteristics of the original thinking from which it draws inspiration.

The work is organised as follows: first, in section 2, we will reconstruct the origin of *mathematical discours* in Poincaré's epistemology, which stems from his criticism of Le Roy. Section 3 will recall the fundamental principles of *discours* in Poincaré's work. The comparison between Poincaré's ideas and modern theory in mathematics education will be the subject of section 4. Finally, section 5 will discuss the explicit and implicit sources in Sfard's works.



## § 2. — Poincaré and Le Roy: the birth of *discours*.

Navigating the monumental work of Henri Poincaré (1854–1912), consisting of more than 500 memoirs and more than 30 volumes, is no easy task, whether it be his scientific output or, even more so, his philosophical writings. In the last ten years of his life, Poincaré developed an epistemological and philosophical framework based on his research in mathematics and physics, which he condensed into four works: *La science et l'hypothèse* (1902), *La valeur de la science* (1905), *Science et méthode* (1908) and *Dernières pensées* (published posthumously in 1913).

Although, as his nephew Pierre Boutroux wrote, 'Henri Poincaré était en philosophie un autodidacte, et il éprouvait à l'égard des systèmes une méfiance particulière'[(2)], 'the ambiguous richness of

[(2)] P. Boutroux, *Henri Poincaré, l'œuvre philosophique*, CreateSpace Independent Publishing Platform, 2016. Source: `https://books.google.it/books/about/Henri_Poincaré.html?id=Kem7jwEACAAJ&redir_esc=y`.

Poincaré's epistemology lies precisely [...] in his participation in the horizon of modernity, while at the same time pointing beyond it'.[3]

In this regard, in the text *La valeur de la science*, it emerges that, starting from an analysis of the state of the art in mathematics and physics research at the beginning of the 20th century, Poincaré establishes his own epistemology by reflecting on the intrinsic value of science and its relationship with reality. The work is divided into three parts: *Mathematical Sciences*, *Physical Sciences* and, finally, *The Objective Value of Science*. It is in the latter that Poincaré distances himself from Le Roy's nominalism[4], according to which the creative power of the scientist is reduced to the invention of a language with which to formulate the laws of nature, which are provisional in themselves as their validity can be disproved at any moment by experience. Poincaré devotes the first chapter of the third part of *La valeur de la science* to Le Roy's philosophy, where he thoroughly analyses the French philosopher's thinking, refuting most of his assumptions. The following passage is particularly interesting for the purposes of this work:

> "Science consists only of conventions, and to this circumstance solely does it owe its apparent certitude; the facts of science and, *a fortiori*, its laws are the artificial work of the scientist; science therefore can teach us nothing of the truth; it can only serve us as a rule of action.
>
> Here we recognise the philosophical theory known under the name of nominalism; all is not false in this theory [...].
>
> That is not all; M. Le Roy's doctrine is not only nominalistic; it has besides another characteristic, which it doubtless owes to M. Bergson, it is anti-intellectualistic. According to M. Le Roy, the intellect deforms all it touches, and that is still more true of its necessary instrument 'discours'. There is reality only in our fugitive and changing impressions, and even this reality, when touched, vanishes."[5]

[3] G. Polizzi, *Henri Poincaré, tra matematica ed epistemologia*, Introduction to *Il valore della scienza*, La Nuova Italia Editrice, Scandicci (Firenze), 1994, p. XVII.

[4] Édouard Louis Emmanuel Julien Le Roy (1870–1954), French philosopher and theologian.

[5] H. Poincaré, *The Value of Science*, Cosimo, Inc., New York, 2007, p. 112.

It is in these lines that the term *discours* appears for the first time in this work, designating *the necessary instrument* of human intelligence in the process of scientific investigation of reality, and not only that, as we shall see later.

Poincaré's introduction of this term in the early stages of his critique of Le Roy's nominalism is the starting point for our comparative analysis with the theory of *commognition*.

## § 3. — *Discours* in Poincaré.

The first question Poincaré asks himself, and which gives the title to the first chapter of the third part of *La valeur de la science*, is whether science is artificial or not. That is, what is the role and purpose of those who do science?

According to Le Roy, science is nothing more than the set of *règles d'action* that scientists apply, even though this does not lead to knowledge of reality, which remains impossible. Poincaré, refuting this view, which Le Roy himself mistakenly accuses of scepticism[(6)], instead highlights the predictive value of these *rules of action*; even if the predictions of science were to be disproved by experience, this would not detract from their value as a tool for knowledge.

On the other hand, Poincaré agrees with Le Roy on the distinction between *fact in the rough* and *scientific fact*: the former is a piece of empirical data, an observable phenomenon not yet subjected to analysis and interpretation, while the latter is a fact in the rough that has been incorporated into a theoretical framework that explains and interprets it through scientific laws. However, Poincaré laments the lack of clarity in defining the boundary between fact in the rough and scientific fact and, since this boundary is drawn by the action of the scientist, it is extremely important to establish it so that the role of those who do science is clearly defined.

> "What difference is there then between the statement of a fact in the rough and the statement of a scientific fact? The same difference as between the statement of the same crude fact in French and in German. The scientific statement is the translation of the crude statement into a

[(6)] É. Le Roy, *Dogme et critique*, Bloud et C$^{ie}$, Paris, 1907.

> language which is distinguished above all from the common German or French because it is spoken by a very much smaller number of people."[7]

In this passage, Poincaré identifies specific language as the tool that translates a crude fact into a scientific fact and emphasises how this specific language is shared by a smaller group of people than those who are able to state a crude fact.

Furthermore, as a predictive tool created by scientists, scientific language will be subject to the necessary changes if its predictions prove to be false. In fact:

> "So that error is the part of man's personal collaboration in the creation of the scientific fact.
>
> But if we can say that the fact in question is false, is this not just because it is not a free and arbitrary creation of our mind, a disguised convention, in which case it would be neither true nor false. And in fact it was verifiable; I had not made the verification, but I could have made it. If I answered amiss, it was because I chose to reply too quickly, without having asked nature, who alone knew the secret.
>
> When, after an experiment, I correct the accidental and systematic errors to bring out the scientific fact, the case is the same; the scientific fact will never be anything but the crude fact translated into another language."[8]

From this perspective, language is not simply a means of expression, but the very condition of scientific knowledge.[9] Poincaré conceives of *mathematical discours* as the epistemic space in which scientific thought takes shape: the truth of science does not lie in isolated facts, but in their linguistic organisation within a shared conventional system. *Discours* thus becomes the tool that guarantees consistency and verifiability, enabling communication and therefore the objectivity of knowledge. In this sense, the cognitive value of science is inseparable from its linguistic structure, which

[7] H. Poincaré, *Op. cit.*, p. 119.

[8] Ibid., p. 120.

[9] F. Furinghetti, *Matematica come processo socioculturale*, IPRASE Trentino, Studi e Ricerche, 2002.

constitutes the very foundation of the scientist's rational and collective activity.[10]

A few lines later, Poincaré adds the adjective *commode* to the noun *langage*; this *langage commode* is the set of terms, laws, postulates, theorems, etc. that will be referred to later with the term *discours*.

It is through language that facts in the rough are transformed into scientific facts, and knowledge is created. Poincaré also turns his attention to what precedes this transformation, namely the fact in the rough:

> "[...], it seems superfluous to investigate whether the fact in the rough is outside of science, because there can neither be science without scientific fact, nor scientific fact without facts in the rough, since the first is only the translation of the second.
>
> And then, has one the right to say that the scientist creates the scientific fact? First of all, he does not create it from nothing, since he makes it with the fact in the rough. Consequently he does not make it freely and *as he chooses*. However able the worker may be, his freedom is always limited by the properties of the raw material on which he works."[11]



Since the scientist cannot influence the facts in the rough, 'all that the scientist creates in a fact is the language in which he expresses it'[12]; he may nevertheless recognise different *realisations* of the fact in the rough, which will contribute to the same scientific fact.

But is the creative work of the scientist sufficient to establish the objective value of science? Poincaré goes on to ask what can be meant by *the objectivity* of science.

In the second paragraph of Chapter XI, *Science and Reality*, we find these crucial lines:

> "What guarantees the objectivity of the world in which we live is that this world is common to us with other thinking beings. Through the communications that we have with other men, we receive from them ready-made

[10] G. Heinzmann & D. Stump (2024), Henri Poincaré. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford Encyclopedia of Philosophy*. Source: `https://plato.stanford.edu/archives/sum2024/entries/poincare/`.

[11] H. Poincaré, *Op. cit.*, p. 121.

[12] Ibid., p. 254.

> reasonings; we know that these reasonings do not come from us and at the same time we recognize in them the work of reasonable beings like ourselves. And as these reasonings appear to fit the world of our sensations, we think we may infer that these reasonable beings have seen the same thing as we; thus, it is we know we have not been dreaming. This, then, is the first condition of objectivity: what is objective must be shared by several minds, and consequently capable of being transmitted from one to another; and since this transmission can only take place through that "discourse" which inspires so much mistrust in M. Le Roy, we are compelled to conclude: No discourse, no objectivity. [...] nothing is objective that cannot be communicated."(13)

> "Nothing, therefore, will have objective value except what is transmissible by 'discours', that is, intelligible."(14)

Poincaré's *mathematical discours*, and above all the language in which it is written, represents a very demanding idea: in order to be the creator of the transition between facts in the rough and scientific facts, the language of mathematics must be rigorous, recognisable to all members of the community that speaks it, and must lead to scientific facts whose predictive capacity can be verified.

## § 4. — ***Discourse*** **in Anna Sfard's theory: points of contact and important differences with** ***discours*** **in Poincaré's epistemology.**

"The meaning of a word is its use in language".(15)

The theory of *commognition* developed by Anna Sfard is an approach that combines the concepts of communication and cognition to explain learning, particularly the learning of mathematics. Sfard proposes that thinking and learning are essentially communicative processes. For an in-depth understanding of the theory of *commognition*, please refer to the seminal work by Anna Sfard(16);

(13) Ibid., pp. 135–136.
(14) Ibid., p. 137.
(15) L. Wittgenstein, *Philosophical Investigations*, Verlag Suhrkamp, Frankfurt, 1953.
(16) A. Sfard, *Thinking as communicating*, Cambridge University Press, 2008.

here we will limit ourselves to outlining the essential points that are relevant to this work.

*Discourse* is defined as any type of communication that brings together certain people (members of *the discourse community*) while excluding others, and mathematical learning takes place by participating in and developing the *discourse* itself.

In fact, Anna Sfard defines the *community of discourse* as follows:

> "A community of discourse is a group of people who communicate according to certain rules and share the same ways of using words, visual mediators, and routines."[17]

> "What distinguishes a community of discourse is not only what its members are talking about, but also how they are doing it."[18]

Sfard argues that *discourse* can be internal (individual thought) or external (interaction with others). This *discourse* develops through language, symbols and other forms of representation, which are all tools of communication.[19]

In particular, mathematics is seen as a special form of *discourse*. Sfard, in fact, considers *mathematical discourse* not only as a means of expression, but as the foundation of mathematical thinking itself.[20] It has a specific structure, which includes rules on how to use symbols, how to formulate propositions, and how to find relationships. For example, the way in which an equation is written or a theorem is proven follows strict rules that are part of *mathematical discourse* involving specific objects, such as numbers, functions, and geometric shapes. These mathematical objects are constructed and manipulated through language and symbolic representations.



[17] A. Sfard, *Learning Mathematics as Developing a Discourse*, Proceedings of the 21st Conference of PME-NA, 2001. Source: `https://www.researchgate.net/publication/252619091_Learning_mathematics_as_developing_a_discourse`.

[18] *Ibid*.

[19] A. Baccaglini-Frank (2021), *To tell a story, you need a protagonist: how dynamic interactive mediators can fulfil this role and foster explorative participation in mathematical discourse*, Educational Studies in Mathematics, vol. 106, Springer. Source: `https://doi.org/10.1007/s10649-020-10009-w`.

[20] A. Baccaglini-Frank, C. Finesilver & M. Tabach (2022), Representations in mathematics education – a shift in perspectives, in *EMS Magazine*, 123, 45–51. Source: `https://doi.org/10.4171/MAG-74`.

In particular, the same mathematical object can have different *realisations*.[21]

Through practice and participation in *mathematical discourse*, abstract thinking, generalisation, and formalisation emerge and develop.

*Mathematical discourse* is neither static nor unique but evolves over time and varies among different *communities of discourse*. This means that, for example, the *mathematical discourse* used in a classroom may be different from that used in a research environment, even though it is part of the same disciplinary field.

The construction and evolution of *mathematical discourse* occurs through interaction between people and mathematical communication through dialogue, both between students and teachers and between peers; this dialogue is essential for learning mathematics.

Students learn mathematics by acquiring the ability to participate in this *discourse*, which includes the use of symbols, diagrams, and specific terms. The way students talk about mathematics reflects and influences the way they think about mathematics.

Learning is therefore defined as the process of becoming competent participants in a *community of discourse*, in this case, the mathematical community. Students learn not only by doing mathematics, but also by participating in the discursive practices that characterise it.

Such participation occurs when students progressively individualise the *discourse* of the expert, adopting its terms, symbols, and language. According to Anna Sfard, students do not simply adopt the tools of the *expert's discourse* but elaborate them to adapt them to their own needs in different contexts. Alongside this communicative relationship, interaction with peers emerges as a further tool capable of developing *mathematical discourse*. Although it is not clearly evident in Sfard's work, we believe that this peer relationship, which is more complex and articulated than that with the teacher, is extremely important, especially for stimulating divergent thinking, the source of scientific creativity:

> "Hardly anyone can understand the importance of an idea, it is so remarkable. Except that, possibly, some children catch on. And when a child catches on to an idea

[21] For example, the stationary point of a quadratic function can have a *graphical* representation (the parabola), an *algebraic* representation (the quadratic function) or a *visual* representation (dynamic geometry files).

> like that, we have a scientist. [...] It's too late for them to get the spirit when they are in our universities, so we must attempt to explain these ideas to children."[22]

Returning to Anna Sfard's thinking, in the learning process, the teacher plays a fundamental role in introducing and guiding students within this *discourse*. Through dialogue, the teacher helps students develop their *mathematical discourse skills*.

In this regard, the theory of *commognition* highlights the gap that can exist between informal and formal *mathematical discourse*, arguing that learning necessarily involves a transition from informal to more *formalised discourse*.

In summary, in her theory, Anna Sfard emphasises that mathematical thinking cannot be separated from language and communication, and that mathematical learning only occurs through participation in *mathematical discourse*. Knowledge, particularly mathematical knowledge, is therefore not a fixed entity, as Sfard points out:

> "Rather than treating knowledge as a static entity, we think of it as something that comes into being through the activity of communication, that is, through discourse."[23]

Anna Sfard attributes to *discourse* the value of a central tool for the creation, communication and transformation of knowledge, especially in the context of science, particularly mathematics. Through *discourse*, knowledge is developed and communicated, linking thinking to the process of communication:

> "Thinking is a form of communication, and learning mathematics means becoming fluent in a certain type of discourse. This discourse is the primary medium through which mathematical knowledge is developed and communicated."[24]

From what has been said so far, both Poincaré and Sfard recognise the central role of *discourse* in the construction of knowledge,

[22] R. P. Feynman, *The Pleasure of Finding Things Out*, Perseus Books, Cambridge, Massachusetts, 1999, pp. 145–146.

[23] A. Sfard, *Learning Mathematics as Developing a Discourse*, Proceedings of 21st Conference of PME-NA, 2001. Source: `https://www.researchgate.net/publication/252619091_Learning_mathematics_as_developing_a_discourse`.

[24] A. Sfard, *Thinking as communicating*, Cambridge University Press, 2008, p. 81.

but it is important to emphasise the different perspectives that characterise their thinking.

In Poincaré, *discours* originates in the scientific community; it develops and evolves as a rigorous tool that allows facts in the rough to be transformed into scientific facts, and it is only thanks to *discours* that science has objective value. In the theory of *commognition*, this concept is extended to any type of communication, both internal and external, and its value as a tool of knowledge transcends science to extend to all forms of learning.

Furthermore, for Poincaré, *discours* translates facts in the rough into scientific facts through the creative work of the scientist, who thus constructs knowledge, whereas for Sfard, *discourse*, in addition to being the means through which learning takes place, is itself the process of structuring thought and is therefore inseparable from the learning process.

It seems to us that a more faithful translation, in terms of learning, of Poincaré's idea, in which language is the tool for moving from raw data to scientific fact, must be achieved by approaching manipulable materials and tools (which are the raw data) since they already belong to science; they already belong to mathematics; not only that, they determine, as essential building blocks, the formal construction that will follow. The manipulation of facts in the rough will also help students to discriminate between those that are useful for verifying a law, i.e. those that can be translated into scientific facts; students will thus learn to exercise what Poincaré calls 'la libre activité du savant'.[25]

## § 5. — Poincaré's works inspiring the theory of *commognition*.

Since her early work in the 1990s, Anna Sfard's reflections have been inspired by Poincaré's thinking. Her desire to investigate the processes underlying learning, particularly the learning of mathematics, stems from the 1908 work *Science et Méthode*, which Sfard quotes repeatedly in her work, where she takes up the French scientist's question:

[25] H. Poincaré, *La valeur de la science*, in *Oeuvres philosophiques de Henri Poincaré*, Édition définitive, Ernest Flammarion, Paris, 1918, p. 255.

> "It is more than eight decades now since the well-known French mathematician and philosopher Henri Poincaré wrote in obvious despair:
>
> *One ... fact must astonish us, or rather would astonish us if we were not too much accustomed to it. How does it happen that there are people who do not understand mathematics? If the science invokes only the rules of logic, those accepted by all well-formed minds ... how does it happen that there are so many people who are entirely impervious to it?*
>
> For all the knowledge accumulated by psychologists and educators since then, this question seems today as challenging and teasing as ever. The particular intricacy of mathematical thinking, the ubiquitous, sometimes insurmountable difficulty experienced by those who learn it, and the resulting persistent lack of success in teaching the subject, all these facts are no less puzzling than they are conspicuous."[26]

This same quotation also appears as the opening words of the first chapter of the monograph *Thinking as communicating*.[27]

The preface to the 2009 edition of this same work reads:

> "Standing on the shoulders of Vygotsky and Wittgenstein, the author defines thinking as a form of communication."[28]

After what has been presented in this work, we believe that Poincaré's name can rightly be added to those of Vygotsky and Wittgenstein. In the French scientist's philosophical writings, we find not only the question that Anna Sfard's entire research work seeks to answer, but above all the recognition of language as a generative structure of scientific knowledge.

*Commognition* takes up this insight, but translates it to a communicative level, where *discourse* becomes the place of production of

[26] A. Sfard, On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin, *Educational Studies in Mathematics*, 22(1):1–36, February 1991. Source: `https://www.researchgate.net/publication/226068580_On_the_dual_nature_of_mathematical_conceptions_Reflections_on_processes_and_objects_as_different_sides_of_the_same_coin`.

[27] A. Sfard, *Thinking as communicating*, Cambridge University Press, 2008, p. 3.

[28] Ibid., front matter.

thought and no longer the condition of its objectivity. In this way, it shifts the emphasis from the epistemological problem (the relationship between language, truth and objectivity) to the pragmatic problem of the shared construction of meaning. However, in doing so, Sfard's theory attenuates the epistemic rigour of Poincaré's *discours*, thus weakening the balance between personal creativity and recognition by the scientific community, which remains one of the most distinctive and fruitful features of the French mathematician's thinking.

Recognising this relationship does not mean interpreting commognition as a continuation of Poincaré's thinking, but rather highlighting a convergence of problems which, although belonging to different contexts, revolve around the epistemological function of language. In Poincaré, discours is the locus of formalisation and rational control; in Sfard, discourse is the realm of interaction and learning. La valeur de la science does not anticipate contemporary thinking, but proposes a different (more rigorous, epistemically binding and still fruitful today) configuration of issues that remain central to the philosophy of science.

Finally, *La valeur de la science* closes with a thought on the creative power of human thought, which we believe can represent, beyond Poincaré's epistemology, the work of those who wonder how to present and teach mathematics to new generations:

> "And yet – a strange contradiction for those who believe in time – geologic history shows us that life is only a short episode between two eternities of death, and that, even in this episode, conscious thought has lasted and will last only a moment. Thought is only a gleam in the midst of a long night.
>
> But it is this gleam which is everything."[29]

[29] H. Poincaré, *The Value of Science*, Cosimo, Inc., New York, 2007, p. 142.

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