

Introduction to the translation of Claude Chevalley's (tentative) Introduction to Bourbaki's *Théorie des ensembles*

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Abstract: We publish, in this special issue of the *Annals of Mathematics and Philosophy* devoted to the notion of proof, an English translation of Bourbaki's *Rédaction N. 066*, a tentative introduction to the *Théorie des ensembles* written around 1948 by Claude Chevalley. This introduction situates the text: its author, the Bourbaki collective and its long effort to axiomatise abstract structures, and what makes this redaction stand out — its philosophical reflection on mathematical knowledge, organised around the idea of proof, and its metamathematical stance.

Keywords: Bourbaki, Chevalley, set theory, foundations of mathematics, proof, truth and meaning, metamathematics, axiomatic method, formalism, structuralism.

We propose, in this special issue of the *Annals of Mathematics and Philosophy* devoted to the notion of proof under the guidance of Brendan Larvor, an English translation of Bourbaki's *Rédaction N. 066*⁽¹⁾ written by Claude Chevalley. The very reason for choosing this text is, besides its intrinsic qualities, its deepness and originality, or its significance for the Philosophy of Mathematics and its History, the fact that it organizes its argumentation starting from the idea of proof: "It is in the nature of a mathematical demonstration to demand the reader's complete and unreserved assent". It then moves progressively to the various implications for the notions of truth and meaning, for the role of words, signs and language or — something Bourbaki's readers would expect — of the axiomatic method and formalism.

Claude Chevalley (1909–1984) was a highly respected, original and influential mathematician. He made fundamental contributions to group theory, algebra and algebraic geometry and published books in these areas that are nowadays considered classics. He taught mathematics at Princeton (1938–1949), Columbia (1949–1957) and Paris (1957–1978).

Born in Johannesburg, son of Abel Chevalley, a diplomat, and Marguerite Sabatier, an English teacher, he entered the École Normale Supérieure in Paris in 1926 where he met Jacques Herbrand, a brilliant young mathematician who was interested in logic and the foundations of mathematics and became a close friend. From 1931 until 1933, he studied in Germany under the guidance of Emil Artin in Hamburg first and then with Helmut Hasse in Marburg. Together with Herbrand, they quickly obtained important results in class field theory. Herbrand died in a tragic climbing accident in 1931.

Chevalley was also known for his manifold interests, including political involvement. The May 1968 student movement challenged his views on education and mathematical elitism profoundly. As a consequence, he helped found the Vincennes University. He also founded the environmental group *Survivre et vivre* in 1970, along with Alexandre Grothendieck and Pierre Samuel. Chevalley's complete works were supposed to bring together mathematics, philosophy, epistemology, and politics, but are only very partially

⁽¹⁾Archives Bourbaki, Donation André Weil (fonds Bourbaki, Archives de l'Académie des sciences). Rédaction N. 066, côte AWR 002, Titre: Livre 1. Théorie des Ensembles. Introduction. 34 p. <https://archives-bourbaki.ahp-numerique.fr/collections/show/3>

published.⁽²⁾ In short, Chevalley, in addition to being a great mathematician, is a multifaceted figure whose wide range of interests is partially reflected in the text we are translating — with, for example, numerous references to literature.

Together with other former students of the École Normale Supérieure — Jean Dieudonné, Henri Cartan, Jean Delsarte, André Weil, René de Possel and, for a brief period, Szolem Mandelbrojt, Jean Coulomb and Charles Ehresmann — he founded Bourbaki in December 1934, the collective who literally changed the face of modern mathematics by publishing a series of treatises in mathematics from the ground up, called *Éléments de mathématique*. The original goal was to write a modern textbook on analysis, from a structuralist point of view, along the lines of Van der Waerden's book on algebra. Depending on how one counts, Bourbaki ended up publishing 30 or 32 books in French, covering a vast amount of what constitutes modern mathematics, and the group is still putting books out.

As we have already mentioned, Bourbaki's goal was to reconstruct analysis from abstract structures, more precisely at the time topological structures, algebraic structures, ordered structures (the "mother structures", in Bourbaki's terminology) and their combinations. These abstract structures are all basically set-theoretical and it made sense, in 1935, to develop an axiomatic set theory in which they would be axiomatically defined. Bourbaki eventually decided to give a general set-theoretical definition of abstract structure, but as one might expect, that enterprise turned out to be more difficult than expected. Indeed, although the first publication on set theory appeared in 1939 — a collection of definitions and results without proofs — the full axiomatic theory and the chapter on structures saw the day respectively in 1953 and 1957! Thus, it basically took twenty years for Bourbaki to come up with a set-theoretical framework and a definition of abstract structure! And in the end, Bourbaki was completely dissatisfied with the published result.

Bourbaki's *modus operandi* has been described in numerous texts. Let us quote for example Cartier's testimony:

Each book [of the series] has been rewritten several times; it is even very difficult to trace its complete history. First, a report was prepared: one of the members

⁽²⁾<https://paysages.math.cnrs.fr/A-la-recherche-des-oeuvres-completes-de-Claude-Chevalley.html>

— usually the one who knew the subject best — was asked to outline the fundamental theories regarding, for example, Lie algebras. In the latter case, it was Chevalley who wrote the report [...]. So, there was an initial report, which we discussed, and from there, we tried to outline what we wanted to present and how to link it to previously published or planned topics. Then, we began the process: there was always a first draft, which the writer would try to develop, and it would come back to the group after six months or a year; we'd discuss it, and we'd be happy with it or not. Generally speaking, we weren't happy with it; it wasn't how we wanted to do it. Sometimes the process could last seven to eight years, with many changes until Dieudonné banged his fist on the table: "That's it, we have to publish." Then he would take all the discussion documents and, in two months, he would synthesize them and bring us the volume as it should be printed.⁽³⁾

Chevalley was asked to write numerous versions of the chapters on set theory and abstract structures. The translated manuscript published here, originally written around 1948, is one of them. Among the numerous attempts made by the various members of Bourbaki, it stands out in two important ways. First, by its intrinsic philosophical content about mathematical knowledge, and, second, its discussion of metamathematics. The latter marks a shift in Bourbaki's attempts at characterizing abstract mathematical structures. Instead of considering the problem as a mathematical question, Chevalley proposes to treat it as a metamathematical challenge. It is certainly relevant to know that the manuscript that precedes Chevalley's chapter in Bourbaki's archive is a very brief account of Gödel's contribution on the consistency of the continuum hypothesis, a purely and remarkable metamathematical prowess, also written by Chevalley. Since he was at Princeton when Gödel lectured on the topic, the chances are he heard about the results and Gödel's views on set theory from early on. He refers explicitly to Gödel's theorems in his manuscript.

⁽³⁾Translated from: Javier Fresán. *Le château des groupes. Entretien avec Pierre Cartier*, Prépublications IHÉS, M/09/41. A partial translation of the interview, *The Castle of Groups. Interview with Pierre Cartier* (that we did not follow as it does not contain the mention of Chevalley we report) is published in the EMS Newsletter, December 2009, pp. 30–33.

Apart from the metamathematical stance, nothing from this manuscript will survive in the published version. Bourbaki will cut all the philosophical discussion in the book on set theory and in the chapter on structures. Technically, Bourbaki will finally release a chapter on structures, knowing full well that their framework cannot, among other things, accommodate categories and functors. Given that in the mid 1950s, they had already published numerous volumes on various mathematical structures, they did not see how they could reboot the whole program. Not to mention the fact that at least two of their members were against treating categories and functors on the same footing as sets.

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